Printed Pages : 3

EAS-103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199109

Roll No. | | | | | | | | | | |

# B. Tech.

## (SEM. I) (ODD SEM.) THEORY EXAMINATION, 2014-15

## MATHEMATICS - I

Time : **3** Hours] [Total Marks : **100**

**Note :** Attempt **all** questions. All questions carry **equal** marks.

1 Attempt any **two** parts :

a) Find the n$^{\text{th}}$ derivative of $\frac{(2x+1)}{(2x-1)(2x+3)}$.

b) If $z = \log\left(e^x + e^y\right)$ then show that $rt - s^{-2} = 0$ where $r = z_{xx}$, $t = z_{yy}$, $s = z_{xy}$. Symbols have their usual meanings.

c) Verify Euler's theorem for function $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$.

2 Attempt any **TWO** parts of the following :

a) Expand $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in ascending powers of $x$.

b) If $u, v, w$ are roots of equation $(x-a)^3+(x-b)^3+(x-c)^3=0$ find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.

c) Using Lagrange's method of Maxima and Minima find the shortest distance from the point $(1,2,-1)$ to sphere $x^2+y^2+z^2=24$.

**3** Attempt any **TWO** parts of following :

a) Find the rank of matrix by reducing to Normal form

$$A=\begin{bmatrix}1 & 2 & 1 & 0\\ -2 & 4 & 3 & 0\\ 1 & 0 & 2 & -8\end{bmatrix}.$$

b) Find the eigen values of corresponding eigen vectors of matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$.

c) If $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$, find $A^{-1}$ and $A^4$ using Caylay's Hamilton's theorem.

**4** Attempt any **TWO** parts of following :

a) Evaluate $\int_0^1\int_0^1 \sin y^2 dydx$ by changing the order of integration.



b) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing into polar coordinates.

c) Prove that $\frac{\beta(p,q+1)}{q}=\frac{\beta(p+1,q)}{p}=\frac{\beta(p,q)}{p+q}$ $(p>0,q>0)$.

**5** Attempt any **TWO** of following :

a) Using Green's theorem, evaluate $\int_c (x^2+xy)dx+(x^2+y^2)dy$ where $c$ is square formed by lines $x=\pm1$, $y=\pm1$.

b) Verify divergence theorem for $\vec{F}=x^3\hat{i}-y^3\hat{j}+z^3\hat{k}$ taken over surface of sphere $x^2+y^2+z^2=a^2$.

c) If $\vec{F}=(x^2+yz)\hat{i}+(y^2+zx)\hat{j}+(z^2+xy)\hat{k}$ find $div\,\vec{F}$ and $curl\,\vec{F}$.

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